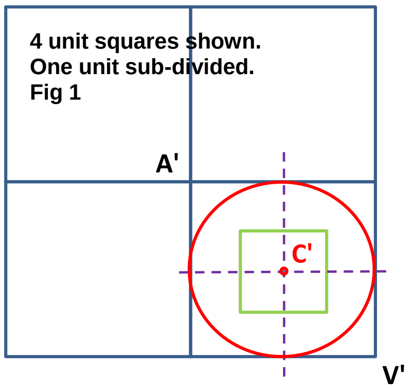


Table 1 - selected results for r=0 to Rmax

0.25 = area of 1/4 of unit square

R	Score area	Prob
0	0	0.0%
0.05	0.0455	18.2%
0.1	0.0821	32.9%
0.15	0.1098	43.9%
0.2	0.1286	51.4%
0.25	0.1384	55.4%
0.26	0.1393	55.7%
0.27	0.1398	55.9%
0.28	0.1400	56.0%
0.29	0.1398	55.9%
0.3	0.1393	55.7%
0.35	0.1313	52.5%
0.4	0.1143	45.7%
0.45	0.0885	35.4%
0.5	0.05365	21.5%
0.525	0.0242	9.7%
0.550	0.0079	3.2%
0.575	0.0007	0.3%
0.5858	0.0000	0.0%

Divide the unit square into 4 equal sub-squares, each of area 1/4. Since there is four-symmetry, the probability is the same for this small square as for the unit square.
Using the 1/4 square was easier for me to visualize.
The score area = area of one-fourth of unit square, less area of quarter-circle of rad R on outer corner, less area of square whose vertex is at center of unit square.

gold score area = 0.25 - π r^2/4 - (0.5- r)^2 for R < = 0.5

Take the derivative, and set to zero to solve for the maximum:

- 2 π r/4 + 1 - 2r = 0. So r = 0.2800

The unit circle, with R = 0.5, has a probability > 0 to land in the square. This is because the center of the r= 0.5 circle can occupy the remanent area in gold of the circle, shown below, without violating the requirements in the problem.

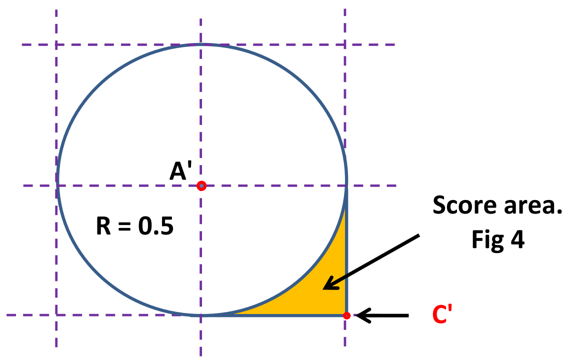
Table 1 shows the overall results

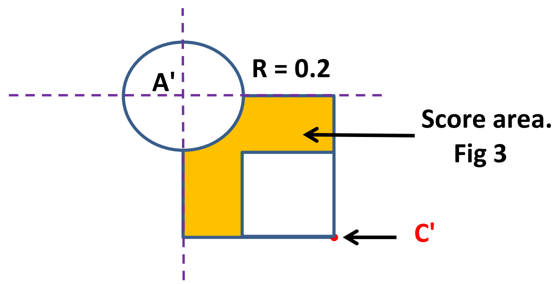
A point (a circle with R=0) will always land in the unit square, so the score area = 0.
Segments in gold are allowable loci of acceptable scores. White segments are forbidden.
Probabilty = score area divided by the area of the quarter-square (which = 0.25).

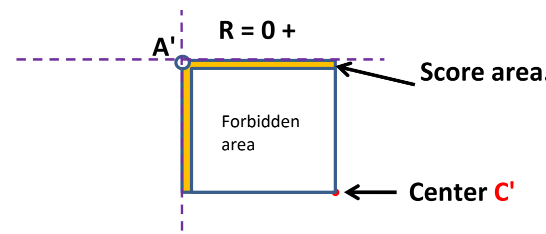
for R> 0.5: green score area = Table 2 below, from 0.5 up to Rmax = 0.5858

Of course, R>0.5 results must match, and they do, the original results at the boundary R = 0.5
Segment in green is the allowable loci of acceptable scores for R > 0.5 to Rmax.

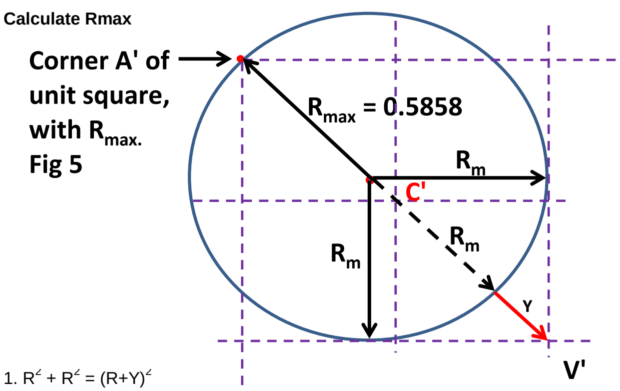
End result: maximum Probabilty of a score = 56% at R = 0.28
It was no surprise that the Probability would decrease as the arcuated triangle got "squeezed" smaller for R > 0.5.







Calculate Rmax



1. $R^2 + R^2 = (R+Y)^2$

Red x-y axes are center of the circle radius R
Blue x-y axes are center of unit square.
Area of kite (ABCD) - area of circle segment (BCD) = green
C = center of red circle
CE = 1-R
ED = x = sqrt (2R-1)
DA = 1 - x - R
AC = (sqrt (2))/2 - [(R-0.5)*sqrt(2)]
CD = BC = R
Use Heron's formula for area of triangle ACD (half the kite):
s = [(DA + AC + CD)/2]
then AREA = sqrt [s*(s-DA)*(s-AC)* (s-CD)]
So green area = 2*AREA - (ϕ/360*πR^2)
where $\phi = 90-2\theta$; where $\theta = \cos^{-1}((1-R)/R)$

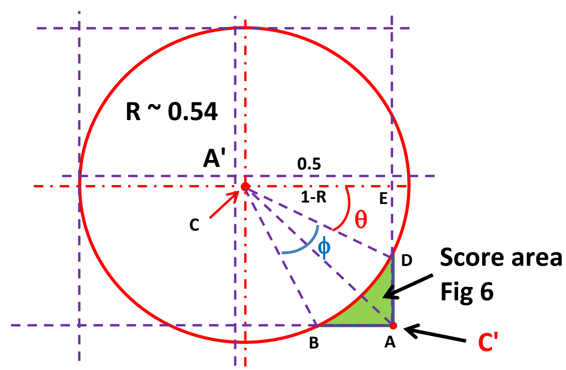


Table 2

R	CE	ED=x	DA	AC	CD	s	AREA	θ	ϕ	ϕ/360*πR^2	green	Prob.