‘Quants’, Kelly, and the RK formula

George Rebane – 19 May 2010 (V5apr18)

[I originally posted this little commentary on some work I had done on the Kelly formula prior to 2010. Well, after some more noodling and squiggly pushing I found that there was a conflict in the way odds and expected gain/loss were defined which required me to revise what is now the RK formula. As a result, I took down the 19may10 version and am now replacing it, appropriately edited and modified where required, with the current corrected version which still reflects its original publishing date.]

Contrary to the rumors you’ve been hearing for the last year plus, GeorgeW didn’t cause the Great Recession. This fiasco had three main progenitors - government idiots, corporate greedies, and some very clever, but not clever enough, Wall Street traders practicing the arcane art and some of the more dubious science of financial engineering.

Scott Patterson has been reporting on things financial for the *WSJ*, and recently wrote *The Quants – How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It* (2010) which immediately shot up to the top of the charts and is still there. I finished *Quants* before going on travel and was just going to add it to my collection of finance and economic history books. But then I thought that there might be some little gems of broader interest there.

Patterson goes back to the beginnings of financial engineering in the 1960s, and takes us right up to the fall of 2009, the Great Recession and all. In fact, the whole purpose of the book is to explicate the role of the quants in fostering the recession. What’s a quant? A quant is a person trained in many of the mathematical ideas and toolsets of the system sciences. A bunch of them came from research universities like MIT, Caltech, UC, … . They are generally at the PhD level, so they have a demonstrated ability to think out of the box and extend human knowledge.

Now these young graduates were snapped up by various ‘Wall Street investment houses’ and hedge funds. Their job was to examine the price behavior of all kinds of securities around the world, and then apply their math wizardry to develop automated trading programs to lickety-split execute profitable arbitrage transactions that their algorithms discovered in the massive daily dataflows of the finance and securities industry. Since I have some knowledge of these high-jinks (actually, I also claim to be a financial engineer), this kind of historical background fascinates me. And doubly so since the political reverberations of this recession will be with us for a long time to come.

Financial engineering actually began with Modern Portfolio Theory credited mainly to Harry Markowitz and William Sharpe. They both became Nobelists for their work. MPT is used to allocate funds among a short list of competing investments like stocks. Sharpe’s contribution was the Capital Asset Pricing Model or CAPM which introduced the notorious alpha and beta numbers to a stock. Brilliant and elegant modeling of an investment world that turned out not to exist – well, not to exist quite like their equations required.

Today MPT and CAPM are used primarily by professionals for CYA purposes to manage other people's money. If the client's portfolio blows up, you can always point to the Nobel prizes and claim you did the prudent thing. And thereby hangs a tale, or should I say 'tail'?.

Most of the applied financial phrenology is still based on the view that security prices move up and down sort of like a drunkard's (or random) walk. If you assume this, you can fit nifty bell curves to the magnitude of price movements, and off you go into the land of stochastics. Stochastics is a fancy word for random processes that depend on other random processes. The problem with the random walk model is that it doesn't predict big catastrophic price swings very well – practically not at all.

Catastrophic price swings have also become known as 'black swans' per the very effective and entertaining evangelism of Nicholas Nassim Taleb familiar to *RR* readers and investors worldwide. When in 2008 things finally went to hell in the markets, everyone knew a black swan had occurred and Taleb was called in to give a few 'I told you so' seminars to some of the snottiest know-it-alls on God's green earth. These, of course, were the quants who were in the process of having a new orifice cut into the south end of their alimentary canals by market price behaviors that weren't supposed to happen.

Patterson's book is a fascinating read on several fronts – investment history, how crazy investment vehicles (don't) work, how to make a LOT of money while still young, and why not to believe in your own press clippings. I recommend it to you.

As an investor and a hopelessly addicted reader, I have quite a library of finance books and have dabbled mightily in the technical side of securities analysis, trading, and portfolio management. I even developed a complete portfolio theory that overcomes the prime sins of the fabled Modern Portfolio Theory – perhaps more on that another time. So I was surprised to discover in Patterson's essay that hedge funds have based the size of their bets on a formula I ran into and expanded some years ago. The formula was developed by communications engineer J.L. Kelly (1956), and is known as the Kelly formula or criterion.

The formula calculates the percentage of your holdings you should bet in each play of a game – i.e. how much should you gamble – given the probability of winning and the payoff odds. Betting the Kelly percentage will then grow your holdings at the maximum rate, while avoiding the notorious 'gambler's ruin', when the policy is used over an extended period. The formula is easy to apply in structured casino gambling situations, but is harder to apply in bets where the odds of winning and payoff are not given. Nevertheless, Warren Buffet and Bill Gross, the 'bond king', among many others, are reported users of this formula. So here it is.

Kelly formula for the optimum fraction f^* to bet

$$f^* = \frac{bp - q}{b} = p - \frac{1-p}{b},$$

$b =$ odds received on wager (e.g. 4:1 is $b = 4$),

$p =$ probability of winning the bet,

$q = 1 - p =$ probability of losing the bet.

Here's an example of its use. Suppose you were playing a game that offers you two to one odds (2:1), and your probability of winning at each play is 49% or 0.49. Then inserting these numbers into the Kelly formula gives f^* = 0.49 – (0.51/2) = 0.235 or 23.5% as the fraction of your holdings that should be bet each time in order to maximize your long-term gain (and minimize the likelihood that you will go bust). Simple.

No matter how favorable the odds b are, the Kelly fraction f^* will always be less than one or 100% as long as your win probability $p < 1$. Kelly advises to bet all your stash only when you are 100% sure that you will win, or when $p = 1$. And, of course, when you obtain $f^* \leq 0$, then you refuse the bet.

But when you bet on a stock and expect to hold it for a time, say six months, you may have in mind a range of prices into which the stock will fall in that time interval. For instance, you may believe that the stock will be somewhere in the range of appreciating 15% to losing 5%. In any event, you don't expect to lose all your money at the end of your six-month bet, as does Kelly with a casino type bet. How do you use Kelly to figure out how much you should bet, er, invest? The answer is you don't because you can't; Kelly doesn't handle this kind of realworld investment problem.

So I started noodling on how to use the same kind of approach that Kelly used, but now on a broader and more useful gambling scenario. Long story short, I was able to derive a generalized formula for handling such a case and, in all humility and giving credit where due, I named it the Rebane-Kelly or RK formula. I even developed a portfolio allocation policy for a short list of securities based on the RK formula, but that's for another time. It's all fun stuff, and I'll now give you the formula, after we first use an example to introduce RK.

Suppose a stock now sells for $100 and you believe that in six months it will be somewhere between $120 and $95, and that's all you can say about it. In other words it is equally likely that the stock will land anywhere in the range of appreciating 20% to a loss of 5% (or an appreciation of -5%).

For this kind of predicted distribution of prices, the expected or average appreciation or gain, given there's a gain, will then be 10% (=20%/2), and its average loss, given that there's a loss, will be 2.5% (=5%/2), both the mid-points of the winning and losing distributions. The probability that the stock will appreciate is then given by the appreciated amount over the range of possibilities calculated as ($120 - $100)/($120 - $95) = 0.80 or 80%, and the probability of loss will be computed from ($100 - $95)/($120 - $95) = 0.20 or 20%, the complement of the probability of gain. (The technical reader will recognize that these calculations result from representing the stock's price performance in terms of a uniform or boxcar probability distribution over its price range.) OK, let's take a look at the RK formula.

The Rebane-Kelly formula for the optimum fraction f_{RK} to invest

$$f_{RK} = \frac{P}{G^-} - \frac{1-P}{G^+},\ \text{(unbounded form)},$$

$P =$ probability of price appreciation or gain,

$G^+ =$ average or expected percent (fractional) gain if stock appreciates,

$G^- =$ average or expected percent (fractional) loss if stock depreciates.

$$f_{RK} = (f_{RK} > 0)(f_{RK} < 1)\, f_{RK} + (f_{RK} \geq 1),\ \text{bounded form so that } f_{RK} \in [0,1].$$

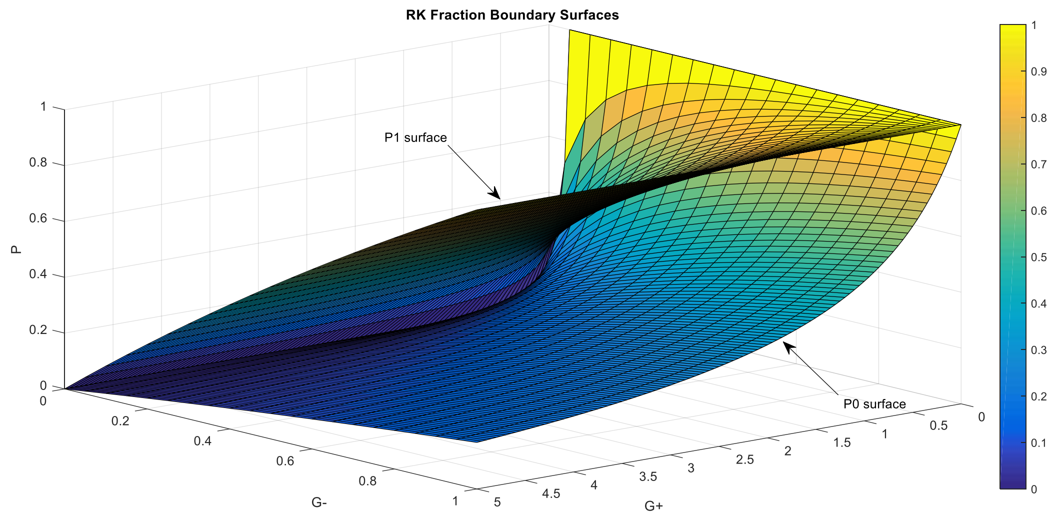
With its logicals that evaluate to either 1 (True) or 0 (False), the last equation above is less complicated than it looks. All it says is that if the unbounded form computes to greater than zero and less than one, then use the unbounded form of the RK formula. If the unbounded form computes to less than or equal to zero, use $f_{RK} = 0$ – i.e. don't bet/invest - and if it computes to greater than one, then use $f_{RK} = 1$ – i.e. invest/bet the whole amount - because those are the limits of the fractional amounts of your available money to invest/bet. The bounded form can be easily entered into an Excel spreadsheet to calculate correct f_{RK} values.

The algebra-savvy reader will note that the Kelly formula is a special case of the RK formula when we set the winning odds to the expected gain, and the expected loss to unity or 100% as occurs in gaming situations.

While the Kelly fraction will never exceed 100%, the unbounded RK fraction will easily calculate over a wide range of negative and positive percentages, that is why the bounded form is presented for the prudent investor. One can argue that obtaining unbounded values above one is a recommendation to use leverage in your investment – i.e. borrow the indicated excess of 100%. It's not clear yet whether a negative unbounded RK fraction recommends selling the investment short.

But we can examine how the RK fraction's input parameter set $\{P,G^+,G^-\}$ yields recommendations in the 0 to 100% range. To see the sensitivity of the RK fraction to its inputs, imagine that the triplet of three parameters represent coordinates in three-space. With a little bit of math we can derive and generate the P1 and P0 bounding surfaces in that space as shown below. As required, the P and G^- axes range between 0 and 1, and the G^+ axis is shown from 0 to 5 or a 500% gain. Only triplets below the upper P1 surface and above the lower P0 surface will yield RK fraction values between 0 and 1 (100%). Points above the upper surface will evaluate to $f_{RK} = 1$, and below the lower surface to $f_{RK} = 0$.

To see how these surfaces work, let's pick the triplet {0.4, 1.5, 0.8} from between the surfaces. Inserting this into the RK formula gives $f_{RK} = 0.20$ or 20%. Picking a point above P1, say {0.7, 0.5, 0.2}, yields the unbounded $f_{RK} = 2.90$ that translates to the bounded $f_{RK} = 1$ or 100%. And picking, say {0.3, 1.0, 0.7}, below P0 yields the unbounded $f_{RK} = -0.27$ that translates to the bounded $f_{RK} = 0$, invest nothing.


RK Fraction Boundary Surfaces
P1 surface
P0 surface
P
G-
G+
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
2.5
3
3.5
4
4.5
5
0.1
0.3
0.5
0.7
0.9