[update] George Rebane, 14 April 2020

So what all should policy makers actually consider as they wrestle with the decision of when and how to reopen the economy? Items definitely to include involve and understanding of the scenarios I have illustrated above regarding second wave infections. (We assume they have access to spread models at least as powerful as Epidyne.) But before that, one needs to take a measure of the situation in answering the question, what is and/or should be the immunity level in the target population I'm considering opening up. From above we saw that the immunity level should be somewhere north of 40% so that herd immunity can kick in and reduce the severity of the second wave after we have again started doing business in the land.

That decision will again involve another round of testing a random sample drawn from such a population. This time the test will be for the presence of immunizing antibodies/antigens, and these tests will also have their own levels reliability expressed in their sensitivities and specificities. More formally these are respectively the probabilities of test positive given the presence of the 'antis', here P^+. and test positive when no 'antis' are present, here P^-. I explained the whole process here, for those needing a review.

From the technical note on population fractions we learned that f_A, the actual or true fraction of the immune in the target population, is related to f_T, the measured fraction of positive test results, by the formula

$$f_A = \frac{f_T - P^+}{P^+ - P^-}$$

If the policy makers decide that they need at least, say, 45% immunity level (f_A = 0.45), then at least what fraction f_T of the administered tests should come back positive? We get the answer by solving the above formula for f_T. This gives

$$f_T = f_A P^+ + (1 - f_A) P^- \ .$$

Suppose we had a hotshot test for the 'antis' with P^+ = 0.98, and P^- = 0.05. The minimum fraction of test positives we demand from the random sample is calculated to be f_T = 0.45*0.95 + (1 - 0.45)*0.05 = 0.469, not much more than what's actually immune in the target population. But if we had a less capable test with, say, P^+ = 0.80, and P^- = 0.10. Then recalculating will prescribe a noticeably lower fraction of positive tests, f_T = 0.415. This is because the worse performing test misses more immune people and therefore undercounts them. Such a result may at first seem counter intuitive, and thinking about is also confounded by the test's false alarm probability P^-. This is especially true when P^- is large, then it tends to compensate for the undercount as seen in the above formula.

Unfortunately, the complexity involved in selecting a threshold f_T for a specified f_A also involves errors in the sampling and also errors in the test's performance parameters - its P^+ and P^- as explained in technical note accompanying the above referenced *RR* post. In our case here, we have to be aware of the error in fT which we use to determine the occurrence of the acceptable level of fA. Fortunately, the error in fT turns out to be simply

$$\sigma_{f_T}^2 = f_A^2\left[\sigma_{P^+}^2 + \sigma_{P^-}^2\right] \rightarrow \sigma_{f_T} = f_A\sqrt{\sigma_{P^+}^2 + \sigma_{P^-}^2}\ ,$$

which in our case becomes $\sigma_{f_T} = 0.45\sqrt{0.005^2 + 0.005^2} = 0.0032$, when we use the test errors called out in the referenced technical note for the C19 infection test. From the gaussian distribution tables this tells us that at a 95% confidence level our tested fraction level should lie between 0.4627 and 0.4753. These error bounds for the fraction of tests positive should satisfy the requirements of policy makers to accept that the target population's actual immunity fraction was within the range to indicate that the second wave of infections would be relatively mild.

Hopefully, this development has given the reader an idea how a reasonable decision can be made in light of concerns about the impact of the second wave after the target population has been given the green light to commence commercial and social activities.