Technical Appendix to 'The real jobs problem – shhh!'

George Rebane – 21 September 2012

From the arguments in 'The real jobs problem – shhh!' let N be the number of workers at a given time, and P ($/worker) be the productivity level or the dollar worth of goods and services produced per worker. Gross Domestic Product is GDP measured in dollars. If these numbers hold for a given year, then we can relate them by $N*P = GDP$.

For the next year let each of these quantities change by respective amounts ΔN, ΔP, and ΔGDP giving us the same relationship which must still hold and now looks like

$$\left(N+\Delta N\right)\left(P+\Delta P\right)=\left(GDP+\Delta GDP\right)$$

It is clear that the fractional rate of change for each of these quantities is calculated by dividing the change by its base amount. We can then solve the above as follows.

$$\text{Let } r_N=\frac{\Delta N}{N}, r_P=\frac{\Delta P}{P}, r_{GDP}=\frac{\Delta GDP}{GDP}, \text{ then}$$

$$N+\Delta N=\frac{GDP+\Delta GDP}{P+\Delta P},$$

$$\frac{N+\Delta N}{N}=1+\frac{\Delta N}{N}=1+r_N=\frac{GDP+\Delta GDP}{N\left(P+\Delta P\right)},$$

$$\text{and recalling that } \frac{GDP}{NP}=\frac{NP}{GDP}=1, \text{ we continue}$$

$$1+r_N=\left(\frac{NP}{GDP}\right)\frac{GDP+\Delta GDP}{N\left(P+\Delta P\right)}=\frac{\frac{GDP+\Delta GDP}{GDP}}{\frac{P+\Delta P}{P}}=\frac{1+r_{GDP}}{1+r_P}, \text{ or}$$

$$1+r_N=\frac{1+r_{GDP}}{1+r_P}.$$

The last equation relating the rates of change can be solved for each rate in terms of the other two, giving

$$r_N=\frac{1+r_{GDP}}{1+r_P}-1,\ r_P=\frac{1+r_{GDP}}{1+r_N}-1,\ r_{GDP}=\left(1+r_N\right)\left(1+r_P\right)-1.$$

These relationships can be easily entered into a spreadsheet to answer any 'what if' questions, and were used to obtain the numbers in the cited post. For example to get the required GDP growth rate that aborbs 1.51% workforce growth in an environment of 3.63% productivity growth, we calculate r_{GDP} as

$$r_{GDP}=\left(1+r_N\right)\left(1+r_P\right)-1=\left(1+0.0151\right)\left(1+0.0376\right)-1=0.0533\simeq 5.3\%.$$