TN1411-1: Gott's future duration, …, and this too shall pass

George Rebane
9 November 2014

[This technical note was initially posted on *Rebane's Ruminations* on 10 November 2014.]

Bet you thought I was going to wax eloquent on some deep philosophical or socio-political issue, a behavior for which I have a weakness as witnessed numerous times in these pages. Fooled you. In another life I am currently working on a couple of very intriguing technical projects involving uncertainty and algorithmics, the kind that I often have to take off at least one shoe in order to get past the sticky parts. One of them involves a nifty and little known method to calculate probabilities about the termination of a kind of realworld and practical processes.

Suppose you live in a metropolitan area where the atmosphere is becoming more polluted by NOX gases as measured by their concentration in the air over the city. And all you know is the record shows that this pollution began about 37 years ago. You need to decide whether to continue living in the city, or to move away, and an important factor in the decision is whether NOX will be brought under control within the next four years. Since this question involves uncertainty, you want to compute the probability that the NOX trend (process) will terminate some time in these four years. And all you know about the process is that it is 37 years old.

Well lucky you, there is a computable answer which turns out to be 0.098, or just under 10% chance that the NOX trend will be brought under control sometime in the next four years.

The solution for such problems comes through a simple formula derived from arguments first presented by physicist J Richard Gott in a quickly forgotten 1993 paper published in *Nature* - 'Implications of the Copernican Principle for our Future Prospects'. In an effort to revive the physicist's thinking, Kierland and Monton made a somewhat cumbersome attempt to explain Gott in their 'How to Predict Future Duration from Present Age' (*Philosophical Quarterly*, 2006). Save for a small flurry of debate, Gott's discovery was peacefully put back to rest.

And then I came along – ta-daa!! Plowing through the paper I was struck by the apparent unrecognized utility of Gott's theory to the analysis of what we may call minimally known processes (MKPs). It was immediately clear to me that in our daily round we are awash in such processes, but very few of us are able to identify them as MKPs, and fewer still have heard of Gott. Since my professional activities continue in various areas of uncertainty, I recognized a diamond in the rough and got to work. My humble contribution from the effort has been a clear derivation of a simple and elegant formula for Gott's probability, that I then extended to support dealing with arbitrary future time intervals, and finally demonstrate the complete scalability of the theory. (For those still awake, we will carry on and promise an intriguing reward to the persistent reader. Nothing beyond clear thinking and the ability to punch numbers into a couple of simple formulas is required.)

Let's open up the horizon with another example modified from Kierland and Monton. If you visited a park in New Zealand's South Island that contains various geysers, and saw from a sign near one geyser informing you that it has been spouting steadily for the last 15,000 years, you could ask for the probability that it would stop doing so within the next couple of hours that you plan to remain in the park. Instinctively, you would conclude that the chances of that process terminating in that short interval would be very low indeed – i.e. the chance of that happening would be somewhere between slim and none (0.000000015).

Walking onward you come upon another geyser encircled by recently erected yellow plastic 'Keep Out' tape, and a nearby sign stating that this geyser started spouting just three days ago. If you now considered the same question, what are the chances that this MKP will terminate in the next two hours, you would naturally conclude that that probability would be much higher (actually 0.027). Well, the Gott theory quantitatively captures those observations in a manner that is totally lucid when examined with the help of the formula we modestly here name Gott-Rebane1 or GR1. Let's take a look.

Say, the MKP in question has been ongoing for a time interval T (years, months, millennia, microseconds, …), and you want to know the probability that the process will terminate in the very next ΔT interval from now, measured in the same units as T. The GR1 formula that gives this probability is

$$P = \frac{\Delta T}{T + \Delta T} \qquad \text{(GR1)}$$

With this you can check my previous answers above, and consider another example. Say that interest rates have been kept near zero for the last five years, and to make some investment decisions you want the probability that this MKP (since all you know about what such interest rate strategy is its current duration) will terminate sometime in the next three months. With your handy calculator you punch in a couple numbers and get

$$P = \frac{\Delta T}{T + \Delta T} = \frac{0.25}{5 + 0.25} = \frac{0.25}{5.25} = 0.048$$

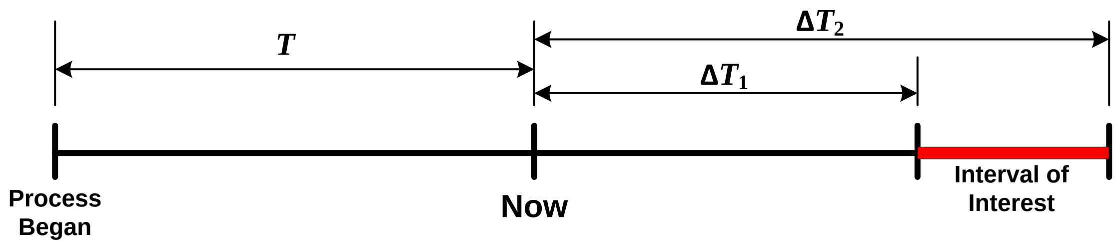
This tells you that the chances of interest rates starting to rise in the next three months is a little less than one out of twenty.

By now fertile minds new to this idea should be coming up with all kinds of examples, perhaps of more interest to them, that can benefit from playing with the GR1 formula given above. But before going off into the wild blue yonder with this newfound ability, be careful that you apply it only to MKPs about which you only know their ages. For example, we can't use GR1 to calculate the probability that Hillary will declare her candidacy sometime during the next X months, even if we knew when she seriously started thinking about entering the 2016 race. And that for two simple reasons – 1) we know that candidates have no impetus to declare their candidacy too early, thereby ending the indecision process prematurely, and 2) we know when the Democratic convention will be along with the November 2016 election, two events that will indeed terminate the process within known bounds. So we can't use GR1 since Hillary's continuing reticence to announce is not a MKP in the sense required by Gott's theory.

Now let's consider an extension of Gott's work. What if the time window of interest was not an interval starting from the present, but instead, say, a six-month period that started nine months from now. What would be the probability that a two year old process would end in that future time window? One can show that such a probability is calculated by the formula we will label GR2 stated as

$$P = \frac{T\left(\Delta T_2 - \Delta T_1\right)}{\left(T + \Delta T_1\right)\left(T + \Delta T_2\right)} \qquad \text{(GR2)}$$

Where ΔT_1 is the time from now until the time window in question starts, and ΔT_2 is time from now that the time window ends. So the numerator is simply the process age multiplying the length of the time window of interest, and the denominator is the product of the process ages to the start and end of the time window. The figure below illustrates this graphically.



Now we can enter in the example problem parameters given above where T = 2 years, ΔT_1 = 0.75 years, and ΔT_2 = 1.25 years.

$$P = \frac{T\left(\Delta T_2 - \Delta T_1\right)}{\left(T + \Delta T_1\right)\left(T + \Delta T_2\right)} = \frac{2\left(1,25 - 0.75_1\right)}{\left(2 + 0.75\right)\left(2 + 1.25\right)} = \frac{2\left(0.50\right)}{\left(2.75\right)\left(3.25\right)} = \frac{1}{8.94} = 0.11$$

So the chances are about one out of nine that the 2-year-old MKP will terminate in the six-month window starting nine months from now.

Note that GR2 becomes GR1 when we let ΔT_1 approach zero. And GR1 behaves as one would intuitively expect if we let ΔT become very large with respect to T – i.e. the interval of interest during which the process can end expands without bound – then P approaches one or certainty. And when ΔT becomes very small with respect to T, then P approaches zero, again as we would expect. There is even more to the mathematics of probability here, but this should be enough for useful and practical back of envelope calculations.

With this under our belt, and with a little more noodling, we can also compute the probabilities of multiple processes starting and ending. Other creative uses of GR2 can be devised that even involve locating things in space.[1] Pretty nifty stuff.

[1] The derivations of GR1 and GR2 are also extended in TN0708-1(10 August 2007) by the author.