TN1808-1: The FD02 Fair Division Algorithm

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For a more complete intro to fair division and its algorithms, please read 'Fair Division 101'. Here we present a brief description of FD02, the multi-party fair division algorithm that can be downloaded from 'Fair Division 101B' on *Rebane's Ruminations*.

The FD02 description refers to the figure below which is snapshot of the MS Excel™ spreadsheet 'FairDivisionAlgo_FD02.xlsx'. The references to the spreadsheet cells follow the column/row convention where, say, F4 denotes the cell in column F and row 4. A range of cells like F14:G19 refer to a rectangular array of cells defined by its opposite corner cells separated by a colon. The blue numbers denote user inputs, and the black numbers are calculated from underlying formulas – don't mess with the black numbers because they are the computed outputs of FD02.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Fair Division Algo FD02**												
2	FairDivisionAlgo_FD02.xlsx												
3	gjr - 5may12 V30aug18												
4	©2018 GJ Rebane				***Party A***	**40%**		***Party B***	**50%**		***Party C***	**10%**	
5				***Asset #***	Appraisal	Bid Value	Received	Appraisal	Bid Value	Received	Appraisal	Bid Value	Received
6				*1*	$ 200	$ 1,000	$ 1,000	$ 220	$ 235	$ -	$ 230	$ 210	$ -
7				*2*	$ 4,000	$ 100	$ -	$ 3,800	$ 3,000	$ -	$ 4,100	$ 4,000	$ 4,000
8				*3*	$ 500	$ 500	$ -	$ 510	$ 550	$ 550	$ 490	$ 450	$ -
9				*4*	$ 1,000	$ 500	$ -	$ 900	$ 500	$ -	$ 800	$ 900	$ 900
10				*5*	$ 2,300	$ 1,200	$ -	$ 2,200	$ 1,500	$ -	$ 2,500	$ 2,400	$ 2,400
11				*6 Cash*	$ 3,000	$ 3,020	$ 3,020	$ 3,000	$ 3,010	$ -	$ 3,000	$ 2,500	$ -
12				***Totals***	$ 11,000	$ 6,320	$ 4,020	$ 10,630	$ 8,795	$ 550	$ 11,120	$ 10,460	$ 7,300
13													
14					A's fair share	$ 2,528		B's fair share	$ 4,398		C's fair share	$ 1,046	
15				Excess/(shortfall) from bidding		$ 1,492			$ (3,848)			$ 6,254	
16				Compensatory cash received/(contributed)			$ (1,492)			$ 3,848			$ (6,254)
17				Bonus Cash received pro rata			$ 1,559			$ 1,949			$ 390
18				Total Value Received			**$ 4,087**			**$ 6,347**			**$ 1,436**
19				Excess Value Received			61.7%			44.3%			37.3%
20				Excess Cash Distributed pro rata	**$ 3,899**								

Suppose three parties A, B, and C must divide up the collection of six assets shown in D6:D11. And the parties agree, perhaps from another agreement or last will and testament, that each is to receive a predefined percentage – shown in F4, I4, L4 - of the total dollar value of the assets. Since, given the diverse assets and each party's individual opportunities and financial situation, they each will almost always have different ideas as to the value of each asset. Perhaps these are obtained from professional appraisers, or simply values the parties designate individually. The spreadsheet allows such values to be entered into the 'Appraisal' columns as shown. These values are there for reference and will not enter into any FD02 calculations.

To implement FD02, all the parties submit bids in dollar amounts that reflect their assessment of the individual asset values. They do this with the knowledge that the highest bidder for any asset will gain ownership of that asset, and that the sum of each party's asset bids reflects that party's total value of the (here six) assets to be fairly divided. In the spreadsheet these bids are shown in

each party's column labeled 'Bid Value' with the totals for each as shown. The submitted bids indicate that Party A will receive assets 1 and 6, Party B gets asset #3, and Party C is awarded assets #2, #4, and #5. The 'Receive' totals reflect what value each party accepts for their assigned assets. Note that asset #6 is $3,000 cash which received premium bids of $3,020 and $3,010 from parties A and B respectively. Why bid more than the cash value itself? Well, it could be that the parties considered garnering the cast at their bid price to be a cheaper alternative to obtaining a loan for some near term expenses such as for the FD02 reconciliation that we will now describe.

Each party's individual fair share of the assets is calculated as the percentage of the total assets each is due. This is simply the product of each party's percentage times his assessed total value of the assets – for Party A this is F4 times F12 and shown as 'A's fair share' in F14. So each party expects his received fair share to be as shown in their respective (I14, L14) cells. We note that these dollar values are determined entirely from each party's own assessments of the assets' values.

Now we bring in the value each received from their bids as shown in cells G12, J12, and M12. This will usually result in either an excess or shortfall of the fair share value they expect, and these are shown in cells F15, I15, and L15. Those parties with an excess will now contribute those excess amounts to a cash pool, and those parties with a shortfall will draw their required amounts to make up the shortfalls from this pool as shown in the 'Compensatory cash…' cells G16, J16, and M16. The result of these contributions and dispersions will always leave an excess cash amount, here shown as $3,899 in cell E20.

The excess cash will then be distributed to the parties pro rata of their accepted share of the assets. This result is shown as the Bonus Cash amounts in cells G17, J17, and M17. Adding each party's received asset values (G12, J12, M12), compensatory receipts/disbursements (G16, J16, M16), bonus cash amounts (G17, J17, M17) gives the dollar sums of their 'Total Value Received' shown in G18, J18, M17. In each case the parties receive more than their own assessed and expected fair share of the divided assets. The percentage of 'Excess Value Received' by each is shown in G19, J19, and M19. And woe be to the party who thinks he can game FD02 to the detriment of any other party – i.e. bidding so as to attempt another party to get less than their individually assessed fair share. Go forth and divide fairly!