TN1902-1: Predicting Termination of Minimally Known Ongoing Processes

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In a 1993 issue of *Nature* physicist J Richard Gott published 'Implications of the Copernican Principle for our Future Prospects', which, despite its convoluted title developed an approach and result for computing the termination probability of a minimally known process (MKP) for which we only know its age T, or the time the process started, but not its total lifetime T_L. From only that data Gott used the Copernican Principle (q.v.) to compute the time boundaries of a future ΔT interval in which the MKP ends with a desired probability P_E. Gott's purpose was to illustrate how the remaining lifetime of humanity could be computed.

The author applied Gott's development to answer the more practical question - given T, what is the probability P_E that the MKP ends during the next ΔT? The solution was developed in [TN0708-1] and, with an alternative derivation, shown to simplify into the elegant

$$\boxed{P_E = \frac{\Delta T}{T + \Delta T} = \frac{h}{1+h}} \tag{1}$$

The result confirms Gott's scalability assertion which is illustrated using the ratio $h = \Delta T / T$.

That development naturally posed the next question – given T, what is the probability P_E that the MKP ends during $\Delta T_{12} = t_2 - t_1$, which begins at some specified future time $t_1 > t_N$, the time now at which the MKP is observed, and terminates at a specified future t_2? The answer to that resulted in the equally fortuitous formula

$$\boxed{P_E\left(T, t_1, t_2\right) = \frac{T\left(\Delta T_2 - \Delta T_1\right)}{\left(T + \Delta T_1\right)\left(T + \Delta T_2\right)} = \frac{h_2 - h_1}{\left(1+h_1\right)\left(1+h_2\right)}} \tag{2}$$

Where $\Delta T_i = t_i - t_N$, the scaling ratios are $h_i = \Delta T_i / T$ for $i = 1,2$, and $\Delta T_{12} = \Delta T_2 - \Delta T_1 = t_2 - t_1$.

Both formulas can be visualized with the help of Figure 1 where (2) reverts to (1) as t_1 approaches t_N and ΔT_{12} becomes ΔT.

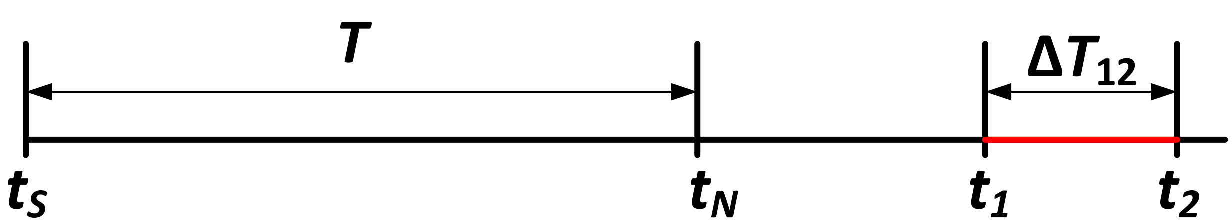

Figure 1

But what if we have a MKP for which we know its lifetime T_L, but not its current age? We then ask the same question – given only T_L, what it the probability P_E that the observed process ends during the next ΔT ? Figure 2 helps us develop the answer. The terms in the figure retain their above definitions.

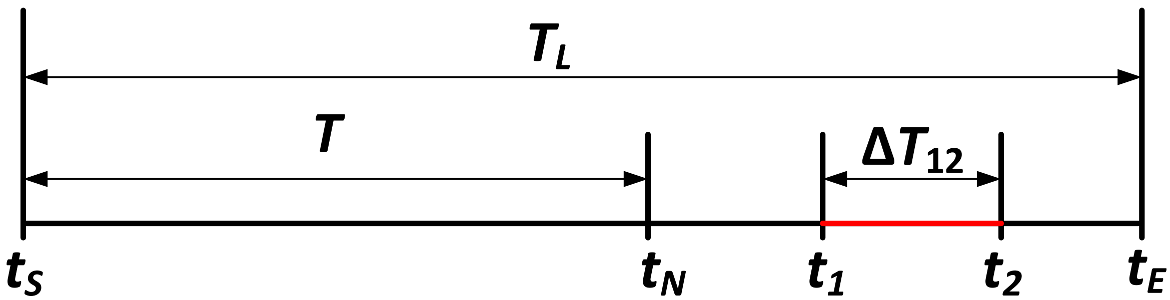

Figure 2

Here we know T_L and ΔT (let $t_1 = t_N$), and now, observing the process at a 'Copernican moment' at t_N , we want to calculate the P_E for the process ending during ΔT, in the red interval $[t_1, t_2]$. Appealing to the Copernican Principle means that t_N may reside anywhere within T_L with equal likelihood. Moreover, if we norm the present time and the process age T within T_L, then we have the uniformly distributed random variable $r = T/T_L \in [0,1]$. And if the process is to end within our time frame of interest, then we must have $T + \Delta T > T_L$, or extend outside the normed interval [0,1]. For the process to survive in the specified ΔT, we must then have $T < T_L - \Delta T$ which occurs with probability P_S where

$$P_S = \frac{T_L - \Delta T}{T_L} \ . \tag{3}$$

Then the probability that the process ends in ΔT is given by

$$P_E = 1 - P_S = 1 - \frac{T_L - \Delta T}{T_L} = \frac{\Delta T}{T_L} \ . \tag{4}$$

Now we again define the scaling ratio $h = \Delta T / T_L$ in terms of the knowns and note that here $h <$ 1. This lets us write the P_E in its generalized, scaled form as

$$\boxed{P_E = \frac{\Delta T}{T_L} = h} \tag{5}$$

Before discussing the ramifications of this result, we extend it to an arbitrary future interval ΔT_{12} as shown in Figure 2. For the process to end in this interval, it must survive until t_1 and then end during ΔT_{12}, or at t_2 at the latest. We write this joint probability as

$$
\begin{aligned}
P_E\left(T_L,\Delta T_{12}\right) &= \Pr\left(\text{process survives until } t_1\right)\Pr\left(\text{process ends } \le t_2 \mid \text{survives until } t_1\right) \\
&= P_S\left(T_L,\Delta T_1\right)P_E\left(T_L-\Delta T_1,\Delta T_{12}\right) \qquad (6) \\
&= \left(1-\frac{\Delta T_1}{T_L}\right)\left(\frac{\Delta T_2-\Delta T_1}{T_L-\Delta T_1}\right) = \left(\frac{T_L-\Delta T_1}{T_L}\right)\left(\frac{\Delta T_2-\Delta T_1}{T_L-\Delta T_1}\right) = \frac{\Delta T_{12}}{T_L}
\end{aligned}
$$

We note that the conditional r.h.s. P_E above accounts for the reduced lifetime given that the process has survived through ΔT_1. Again we define the respective scaling ratios as $h_i = \Delta T_i / T_L$ for i = 1,2. This lets us express (6) in its generalized, scaled form as

$$
\boxed{P_E\left(T_L,\Delta T_{12}\right) = \frac{\Delta T_{12}}{T_L} = h_2 - h_1} \qquad (7)
$$

When comparing answers to the above questions we asked about these two MKPs, we immediately note that when only the process lifetime T_L is known, then the desired termination probability P_E is independent of when the interval of interest occurs, and dependent only on the duration or length of the observation time interval. However, when only the MKP's age T is known, then the P_E results given in (1) and (2) are seen to depend not only on the duration of specified future interval, but also how far in the future that interval occurs.

Both results are readily intuited by appealing to the Copernican Principle in that the current observation time t_N is not privileged in any manner, and in the first case may have occurred at the end of age T with equal likelihood any time within the process lifetime (as originally posed by Gott), and in the second case with equal likelihood any time within known lifetime T_L, as posed above.

In the case of the unknown T_L, P_E decreases with a constant duration ΔT_{12} pushed further into the future because 1) process survival probability P_S decreases over longer future intervals, and 2) the P_E for the same duration ΔT_{12} starting in the future will be lower because the process will then have aged more. Equation (2) accounts for both of these factors and TN0708-1 has the details.

Applications of MKP termination probabilities

Both kinds of problems – only T_L or T known – yield termination probabilities that are useful in themselves, but are also useful as prior probabilities in a Bayesian analysis that incorporates new evidence about the ongoing process. For example, to estimate the change in state (e.g. breakdown) of piece of equipment in operation for a known time, P_E from (1) or (2) can be used to support decisions on when/whether to replace it in some future time interval, or when it may be prudent to do some preventive maintenance on it. Equations (1) and (2) are also useful for calculating the probability that some previously announced event will happen within a specified time interval. In such problem formulations the ongoing 'process' is the ongoing delay in, say, a candidate's announcement of the anticipated decision to run or not. When MKPs are expanded to include such phenomena, the diverse uses will be constrained only by the creative way they are applied to realworld problems, and, of course, being careful to meticulously apply the minimum knowledge criterion.

When only T_L is known about an ongoing process of interest, then (5) and (7) can be used for the same previously described purposes. If only a known observation interval is possible, then for such processes it does not matter when that observation interval is applied for the purpose of witnessing or monitoring or measuring the termination event. The T_L version is specifically useful for limited observations of periodic processes in which each period can be considered as a process. For example, if one can only observe a multi-lap bicycle race for a fraction of a lap time (ΔT_{12}) in order to, say, see a favorite rider pass by, then it doesn't matter when you arrive during the race to start your allocated interval. In this case the state of the 'process' is your favorite rider's location on the lap's circuit, which you don't know, but do know the time it takes riders to complete a lap. Only the time you are willing to spend observing will affect P_E, your probability of seeing the rider. The same kind of analysis can be applied to reserving time on a valuable instrument, say, a telescope to observe a periodic process such as an exo-planet crossing its sun. Again, creativity is key to identifying such applications that satisfy the minimum knowledge constraint.